Indian Statistical Institute, Bangalore
B. Math (II)
Second semester 2003-2004
Backpaper Examination : Statistics (II)

Date: 21-07-2004 **Maximum Score 50** **Duration: 3 Hours**

1. A manufacturing unit produces digital thermometers. Suppose X denotes the lifetime of a digital thermometer. Suppose also that the lifetime of each thermometer, measured in hours, has *exponential distribution* with *pdf* $\lambda e^{-\lambda x} I_{(0,\infty)}(x)$. Suppose $X_1,\ X_2, ..., X_n$ denote the lifetimes of n randomly selected thermometers produced in the manufacturing unit.

   (a) Show that $T = \Sigma_{i=1}^{n} X_i$ is *minimal sufficient* for λ.

   (b) Obtain a *UMVU* estimator for λ.

   (c) Does the variance of your estimator attain *CRLB*? Comment.

   (d) Find $E(T^k)$, k being an integer such that $n + k > 0$. [11]

2. Bala buys a liter of milk everyday. Suppose his consumption X of milk measured in liter has the following *distribution* with *pdf* $\theta x^{\theta-1} I_{(0,1)}(x)\ ; \theta > 0$. Let $X_1,\ X_2, ..., X_n$ denote the consumption of milk on n randomly chosen days. Bala has reasons to believe that θ has a *prior distribution* given by $Gamma(a, b)$, $a > 0$, $b > 0$ known.

   (a) Suggest a *method of moments estimator* for θ based on the random sample $X_1,\ X_2, ..., X_n$.

   (b) Obtain *posterior distribution* of θ given the observations $X_1,\ X_2, ..., X_n$. Obtain mean and variance of the *posterior distribution* of θ.

   (c) Suggest *Bayes estimator* for θ. [11]

3. Let X_n, Y_n, $n \geq 1$ be sequences of random variables and X be a random variable, all defined on the same probability space, such that $X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{p} c$, where c is a finite constant. Suppose $g : \mathbb{R} \longrightarrow \mathbb{R}$ is a continuous function and $a, b \in \mathbb{R}$ then prove that $g(X_n) + aY_n + b \xrightarrow{d} g(X) + ac + b$. [11]

4. A machine produces metal rods produced in automobile suspension system. A random sample of 12 rods is selected and the diameter of each of the rods is measured. The resulting data in *mm* are shown here:

8.23	8.31	8.42	8.29	8.19	8.24
8.19	8.29	8.30	8.14	8.32	8.40

   The machine is supposed to be producing rods of diameter 8.20 *mm*.

   (a) List carefully the assumptions you must make and formulate the problem of testing of hypotheses for the problem mentioned below in (b).

   (b) Is there evidence to indicate that mean rod diameter is different from 8.20 mm at 5% level of significance.

   (c) Find the *p-value* of the test.

   (d) Find 90% *confidence interval* for the expected rod diameter. [11]

5. A semiconductor manufacturer produces controllers used in automobile engine applications. The customer requires that the process fallout or the fraction defective at a critical manufacturing step not exceed 0.05 and that the manufacturer demonstrate process capability at this level of quality using $\alpha = 0.05$. The semiconductor manufacturer takes a random sample of 200 devices and finds that 4 of them are defective. Can the manufacturer demonstrate the process capability to the customer. Let θ be the probability of a device being defective.

   (a) List carefully the assumptions you must make to formulate the problem of testing of hypotheses $H_0 : \theta \leq 0.05$ versus $H_1 : \theta > 0.05$

   (b) Carry out a test at 5% level of significance.

   (c) Find the *p-value* of the test.

   (d) Find 90% *confidence interval* for the fraction of defective devices produced. [11]